Bachelor of Science (B.Sc.) Semester—II (C.B.S.) Examination
STATISTICS
(Probability Distributions)
Compulsory Paper—1

Time : Three Hours]　　　　　　　　　　　　　　　　　　　[Maximum Marks : 50

N.B. :— All questions are compulsory and carry equal marks.

1. (A) Define a Bernoulli trial. State the p.m.f. of Bernoulli distribution. Obtain its m.g.f. and hence find its mean and variance. State the probability distribution which is generalization of Bernoulli distribution. Find its p.g.f. Hence find its mean and variance. 　10

OR

(E) If a r.v.x follows Poisson distribution with parameter λ, obtain its first four raw moments about origin. Hence find its first four central moments. Compute β_1 and β_2 and comment on skewness and kurtosis. 　10

2. (A) Derive the p.m.f. of a r.v. which follows Geometric distribution. Why is this distribution called as 'Geometric' distribution ? Obtain its moment generating function and hence find its mean and variance. State and prove lack of memory property of Geometric distribution. 　10

OR

(E) Derive the probability mass function of hypergeometric distribution. Find the mean of this distribution.

(F) (i) The probability that a student pilot passes the written test for a private pilot's license is 0.7. Find the probability that the student will pass the test (a) on the third try (b) before the fourth try.

(ii) The probability that a mouse is infected with a disease is $\frac{1}{6}$. A scientist innoculates the mice until he finds second infected mice. Find the probability that the scientist stops innoculation after inspecting 8 mice. 　5+5

3. (A) Derive the moment generating function of a Normal distribution. Obtain the Mean and Mode of this distribution. Show that a linear combination of independent normal variables is also a normal variable. 　10

OR

(E) Define a standard normal variable. Obtain its probability density function.

(F) Obtain the median of Normal distribution.

(G) If X denotes the number of points obtained in a single throw of an unbiased die, write p.m.f. of X and find its mean and variance.

(H) For a rectangular distribution $f(x) = \dfrac{1}{2a}, -a < x < a = 0$ elsewhere,

Obtain moment generating function about origin and show that odd ordered moments about origin are zero for this distribution.   2½×4=10

4. (A) Define beta variate of first kind. Obtain an expression for rth moment about origin for it. Hence find its mean and variance.

(B) Write the probability density function of Gamma distribution with one parameter and two parameters. When does gamma distribution with two parameters reduce to gamma distribution with one parameter ? State and prove additive property of gamma distribution with one parameter.

5+5

OR

(E) Obtain the moment generating function of random variable X which follows exponential distribution with parameter θ.

(F) Show that mean is equal to variance for gamma distribution with one parameter.

(G) Find harmonic mean of beta distribution of second kind.

(H) If a r.v. X follows beta distribution of second kind then find its rth moment about origin.

2½×4=10

5. Solve any **ten** of the following :

(A) If X ~ B(n, p). State the probability distribution of (n – x).

(B) The m.g.f. of a discrete r.v. X. is $e^{4(e^t-1)}$. Write V(x) and third central moment of X.

(C) State the conditions under which Binomial distribution tends to Poisson distribution.

(D) Name the appropriate probability distribution for the number of interviews that would have to be conducted to get the first acceptable candidate.

(E) Find the parameters of the Negative Binomial distribution whose mean is 8 and variance is 16.

(F) If the c.d.f. of a r.v. X is given by

$\quad$ F(x) = 1 – e$^{-\theta x}$, x > 0

$\qquad$ = 0 $\qquad$, x ≤ 0

and θ > 0, find p.d.f. of the r.v. X.

(G) If the r.v. X follows Gamma (3, 4), then E(X) = _____ and V(X) = _____.

(H) Calculate the mean of exponential distribution with parameter 5.

(I) If P[Z < 1.65] = 0.95 then find P[Z < –1.65] where Z is the standard normal variate.

(J) Write the m.g.f. of standard normal variate.

(K) If X is the discrete uniform r.v. taking values 1, 2, ------, N, then find its mean.

(L) State the m.g.f. of negative binomial distribution.   1×10=10